



# A New Ice-sheet / Ocean Interaction Model for Greenland Fjords using High-Order Discontinuous Galerkin Method



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The goal of this project is to build a separate, high-resolution module for use in Earth System Models to realistically represent the fjord bathymetry, coastlines, exchanges with the outside ocean, circulation and fine-scale processes occurring within the fjord and interactions at the ice shelf interface, using discontinuous Galerkin (DG) method.

## Motivation

- Ice-sheet/ocean interaction in narrow fjords around Greenland is one of the key outstanding challenges in modeling studies of climate change and sea level rise
- The range of geometry scales varies greatly from thousands of kilometers in regional scale down to single kilometers within the fjords.
- Current models are unable to resolve fine-scale processes in the fjords without prohibitive computational cost

## Main features of the module

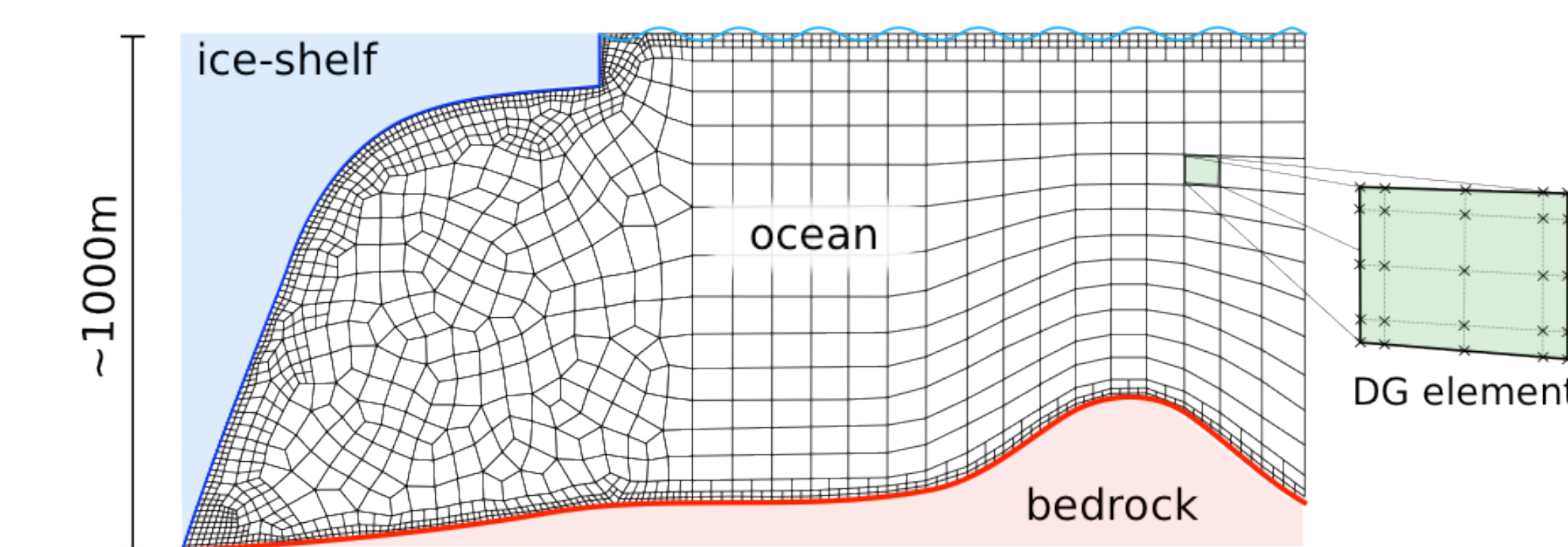
- **Non-hydrostatic** DG model for incompressible Navier-Stokes
- Accounts for the stationary ice-shelf with sub-shelf ocean interaction, basal melting and subglacial meltwater influx
- Designed to two-way **couple with Earth System Models** through a standard CPL7 coupler
- Uses local non-conforming mesh refinement to **resolve small-scale features** and avoids overwhelming computational cost

## Sermilik Fjord and Helheim glacier

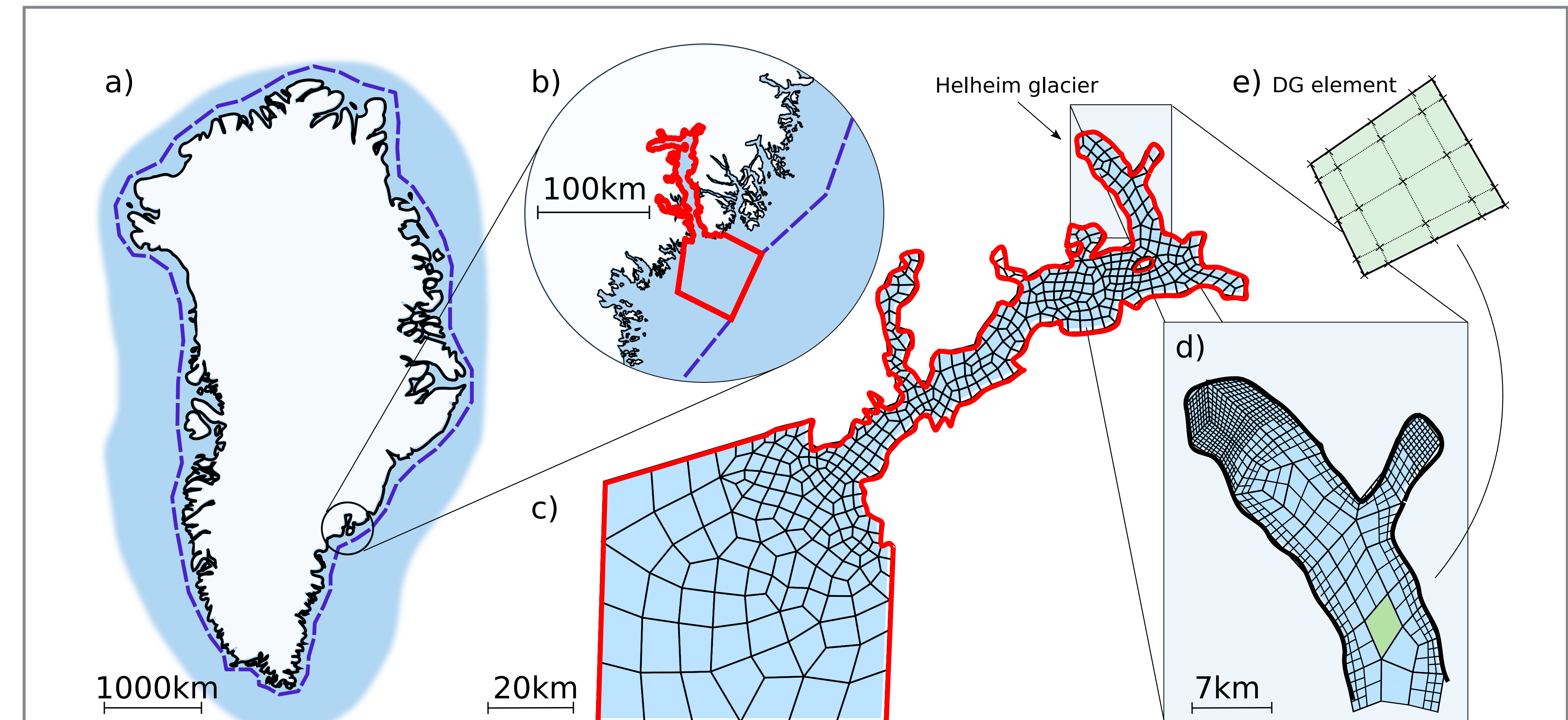


This system is a testbed for the new ice-sheet / ocean interaction model. The model will be set up using measurements of CTD, bathymetry, glacier front position, subglacial melt rates, etc. Contour of the domain generated using the GSHHS coastline dataset is overlaid with Google Maps satellite image.

## Unstructured mesh under the ice-shelf



- Smooth representation of bathymetry and ice-shelf geometry
- Zero-thickness water column at grounding line, good representation of subglacial discharge
- Local refinement provides high resolution near the turbulent boundary layer without significantly increasing the cost

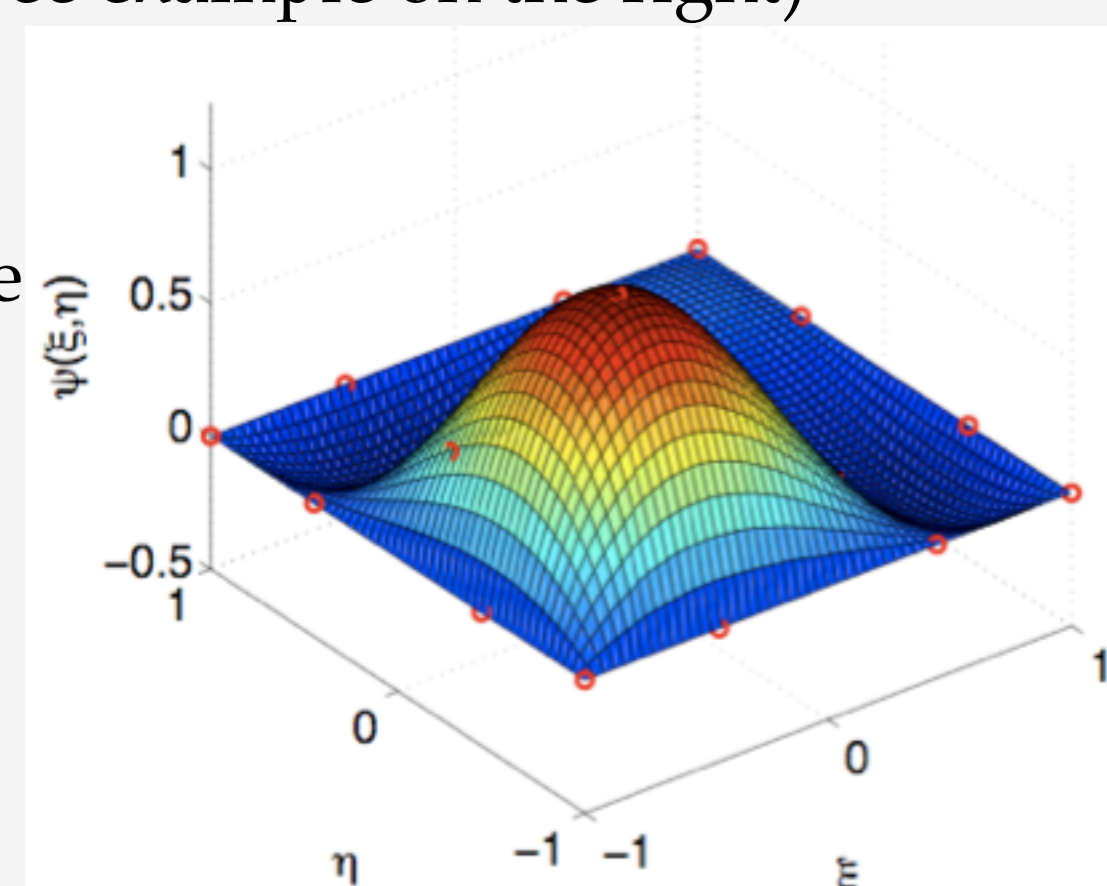


## Hierarchical approach to modeling the ice sheet-ocean interactions in the coast of Greenland.

- (a) The largest scale domain envelopes Greenland and interfaces small-scale features of the fjord flow with the general circulation ocean model.
- (b) In order to demonstrate the feasibility of this approach we initially focus on the Sermilik Fjord
- (c) The quadrilateral element mesh represents the geometry of the fjord accurately and allows for further non-conforming refinement
- (d) Non-conforming mesh refinement permits recursive subdivision of the elements in the deep end of the fjord in order to increase the resolution near the ice-sheet.
- (e) The solution in each element is expanded using a high order DG method, which allows for an accurate representation of small-scale features.

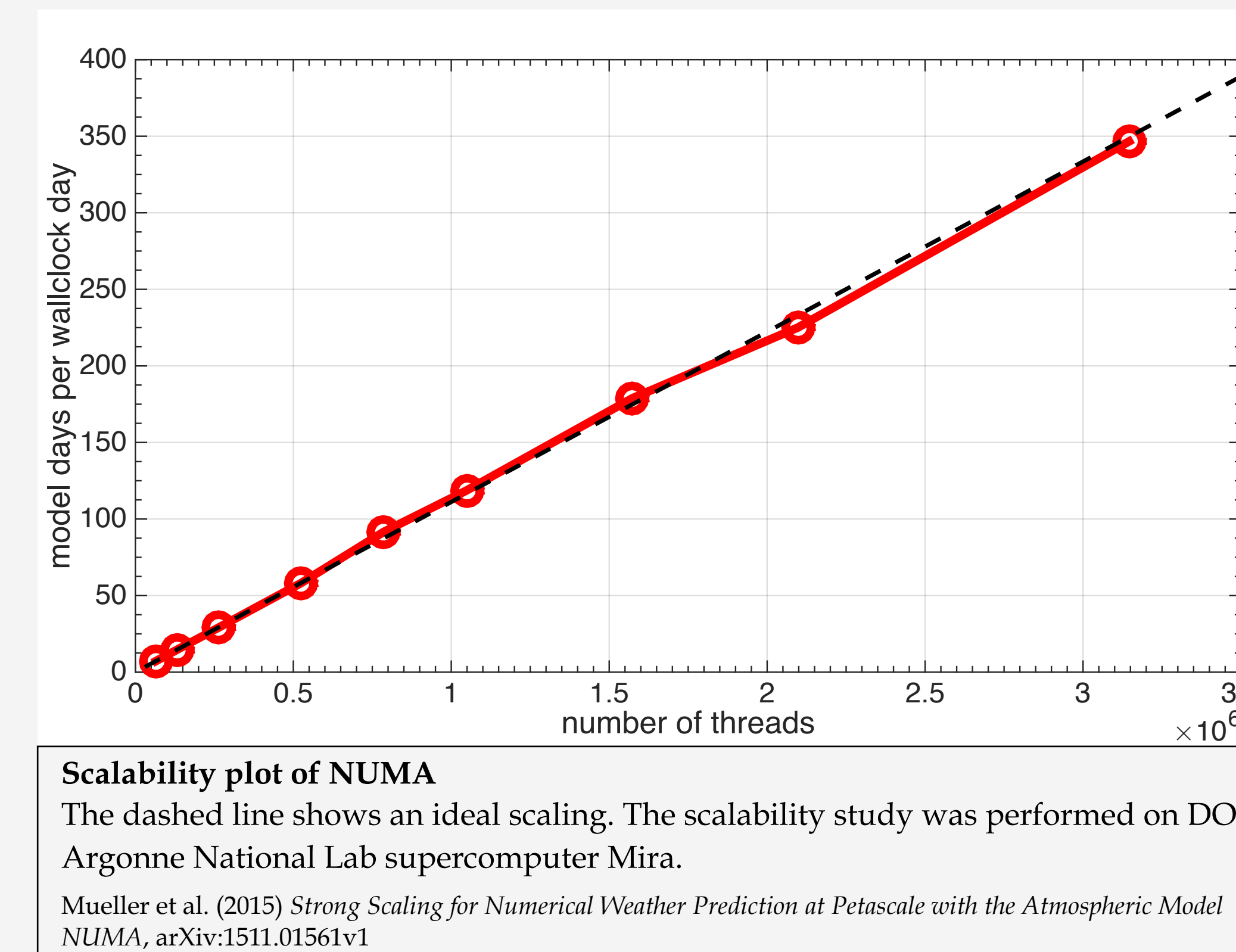
## Discontinuous Galerkin method

- Domain is decomposed into a mesh of unstructured elements
- Inside the elements the solution is expanded in a base of high-degree polynomials (see example on the right)
- The governing equations are solved in each of the elements separately and are coupled between the elements by exchanging fluxes.
- An unstructured element mesh provides excellent geometrical flexibility
- DG naturally allows for non-conforming refinement and is mass conservative



## Scalability of the DG method

- By organizing computations within the elements the method achieves high computational intensity, while small communication stencils (nearest face-neighboring elements only) leads to excellent scalability
- Figure (right) shows 99% efficient strong scaling up to **3.14 million threads** (786,432 cores, 4 independent threads each) of an atmospheric model NUMA using **1.8 billion points** and continuous Galerkin method. DG is expected to scale the same or better due to smaller communication stencil.
- This scalability enabled NUMA to run 3km global resolution simulation in time feasible for operational weather prediction (1 model day in 4.5 minutes simulation or 320 model days per wall clock day). The model achieved **1.2 petaflops** in most computationally intensive subroutines.



**Scalability plot of NUMA**  
The dashed line shows an ideal scaling. The scalability study was performed on DOE Argonne National Lab supercomputer Mira.  
Mueller et al. (2015) *Strong Scaling for Numerical Weather Prediction at Petascale with the Atmospheric Model NUMA*, arXiv:1511.01561v1

## Other applications of DG

- Introduced in 70's for neutron transport (Reed and Hill 1973)
- Used in ocean and atmospheric simulations for:
  - shallow water flows, tsunamis (e.g. Schwanenberg et al. 2000, Giraldo et al. 2002, Nair et al. 2005, Blaise and St-Cyr 2012)
  - 3D hydrostatic ocean models (Aizinger and Dawson 2007)
  - 3D non-hydrostatic atmospheric model (Kelly and Giraldo 2012)

## References

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