Physics-dynamics coupling with Galerkin methods: equal-area physics grid

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Introduction

Consider a cubed-sphere tiling of the sphere with quadratic elements on each face. Inside each element there are 4x4 Gauss-Lobatto-Legendre (GLL) quadrature points:

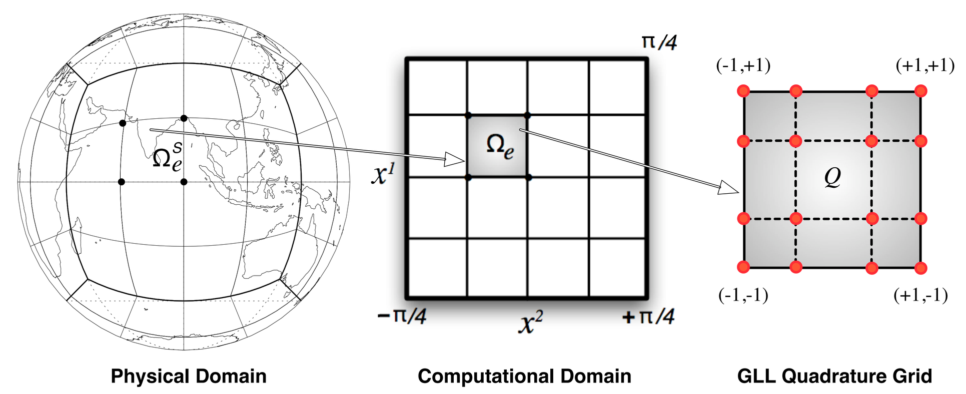


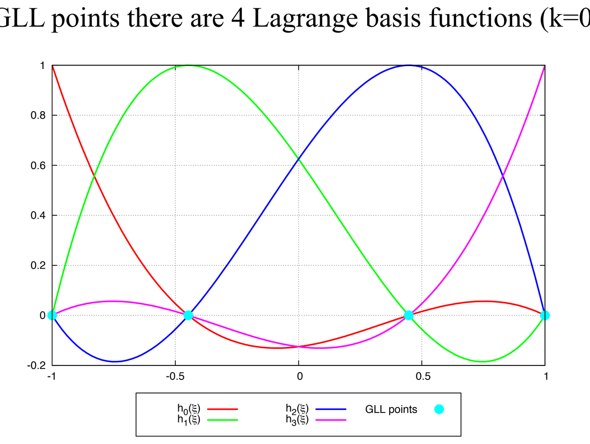
Fig. 9.22 A schematic diagram showing the mapping between each spherical tile (element) Ω_e^S of the physical domain (cubed-sphere) $\mathscr{S}$ onto a planar element Ω_e on the computational domain $\mathscr{C}$ (cube). For a DG discretization each element on the cube is further mapped onto a unique reference element Q, which is defined by the Gauss-Lobatto-Legendre (GLL) quadrature points. The horizontal discretization of the HOMME dynamical cores relies on this grid system.

(Figure and caption from Nair et al., 2011)

Assume a nodal basis set constructed using Lagrange polynomials $h_k(\xi)$, ξ=[-1,1]:

$$h_k(\xi) = \frac{(\xi-1)(\xi+1)P_N'(\xi)}{N(N+1)P_N(\xi_k)(\xi-\xi_k)},$$

where $P_N(\xi)$ is the Legendre polynomial of degree N and $P'_N(\xi)$ is the derivative of $P_N(\xi)$. With 4 GLL points there are 4 Lagrange basis functions (k=0,1,2,3):

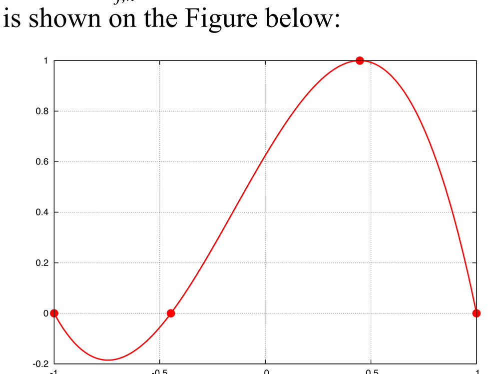


The solution U at time t inside element j is given by

$$U_j(\xi,t) = \sum_{k=0}^{N} U_{j,k}(t)\,h_k(\xi), \quad \xi \in [-1,1],$$

where $U_{j,k}(t)$ is the known value at the k^{th} GLL point. Note that the solution is expressed as a Lagrange interpolation polynomial.

Given GLL point values, $U_{j,k}(t) = \{0,0,1,0\}$ for k=0,..,3, the Lagrange "reconstruction" is shown on the Figure below:



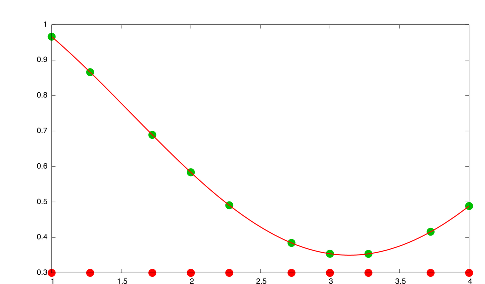
For simplicity we show only 1D examples; the 2D basis set can be constructed with a tensor product of the 1D basis functions:

$$U_h(\xi,\eta,t) = \sum_{\ell=0}^{N}\sum_{m=0}^{N} U_{\ell m}(t)\,h_\ell(\xi)\,h_m(\eta), \quad \text{for} \quad -1 \le \xi,\eta \le 1,$$

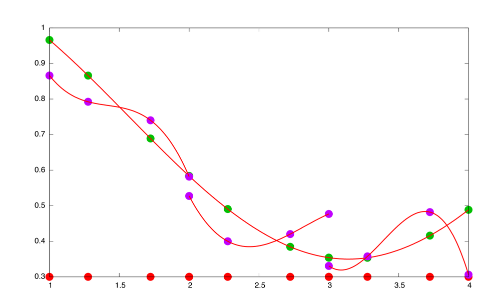
Physics-dynamics workflow

Consider the continuous Galerkin finite-element method used in CAM-SE (NCAR's Community Atmosphere Model – Spectral Elements).

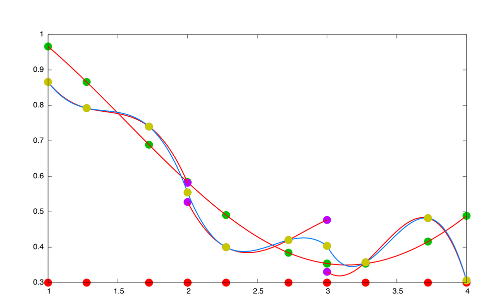
For simplicity consider a domain of 3 elements in 1D and let the initial condition be a "global" degree 3 polynomial (which can be represented exactly by the polynomial basis). Note that GLL points at element edges are shared between neighboring elements:



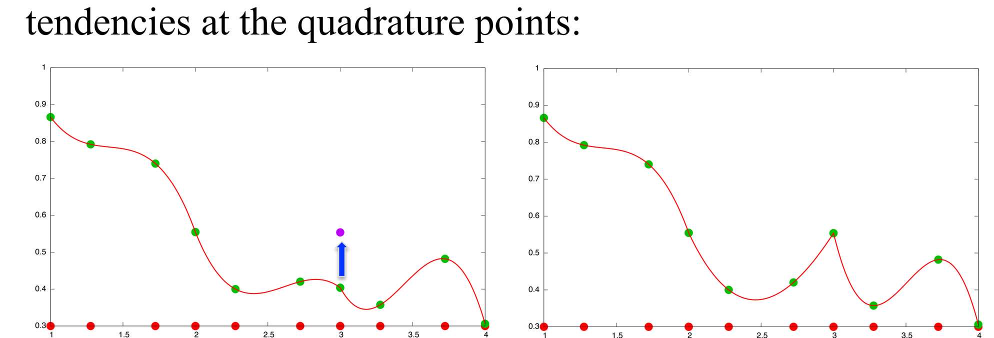
The solution is advanced one Runga-Kutta step inside each element:



The solution is projected onto a C^0 basis (GLL point values at element edges are averaged – blue curve below):



This process is repeated for each Runga-Kutta step. Now the physical parameterization suite is called which, based on the atmospheric state at the GLL point values, computes tendencies at the quadrature points:



Assume that there is only a physics update for the GLL point located at x=3 (see left Figure above). After physics has updated the atmospheric state at the GLL point(s), the polynomial "reconstruction" is shown on the Figure to the right (above).

Note that the solution is only C^0 at element boundaries! This is typically where noise appears!

Grid-scale forcing and noise

The spectral-element "reconstruction" is least smooth at the element boundaries where the C^0 constraint is enforced; in climate simulation with CAM-SE noise in topographically forced flow typically appears near element boundaries (see Figures below).

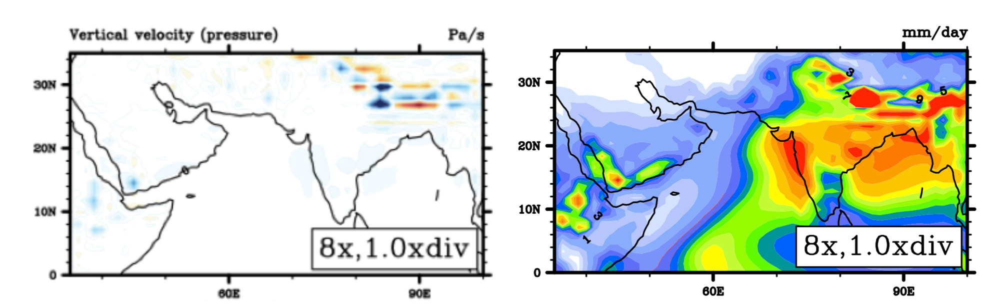
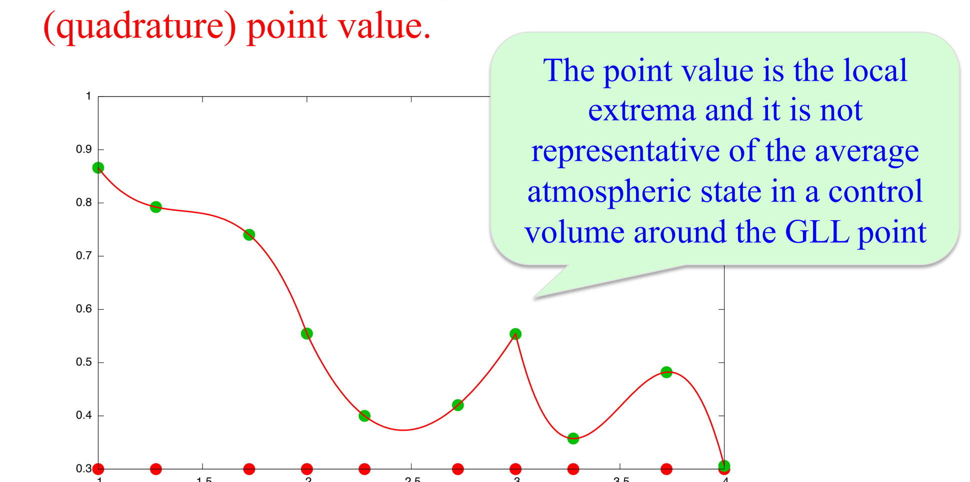


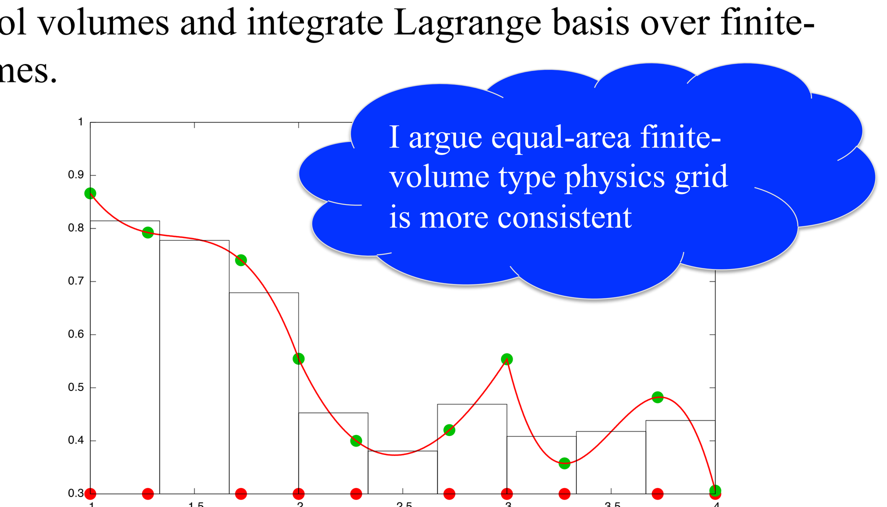
Figure: *(left) 30 year average vertical pressure velocity for AMIP run using rough topography and no extra divergence damping. (right) Same as (left) but for precipitation rate.*

State from dynamical core passed to physics

I argue that parameterizations should be given a grid cell mean value for the atmospheric state rather than a (quadrature) point value.



Definition of physics grid: Define equal-area physics grid in each element by dividing each element into equi-distant control volumes and integrate Lagrange basis over finite-volumes.



Note that physics grid averages/moves fields away from boundary of elements where the solution is least smooth (in element interior the polynomials are C^∞)

Held-Suarez forcing with "real-world" mountains

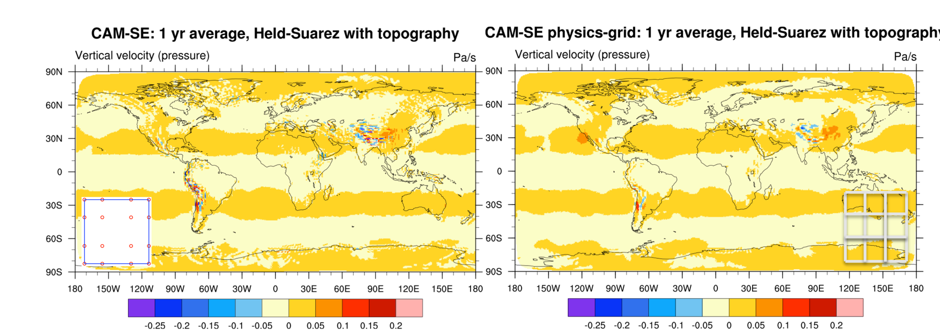


FIGURE 10. One-year average of vertical velocity (ω) using Held-Suarez forcing and 'real-world' topography using CAM-SE at approximately 2° horizontal resolution ($ne16np4$). Left plot is based standard CAM-SE setting where the sub-grid scale parameterization are computed on the spectral element quadrature grid and the right plot is based on the physics grid version in which tendencies are computed on a $3x3$ finite-volume grid inside each element. Note that the physics grid has the same number of degrees of freedom as the quadrature grid in this configuration.

Note: in this experiment bilinear interpolation was used for moving variables to and from physics-dynamics grid.

Transferring variables from physics grid to dynamics grid

Moving variables from dynamics to physics through basis function integration is likely the most consistent/accurate approach; going the other way is less obvious:

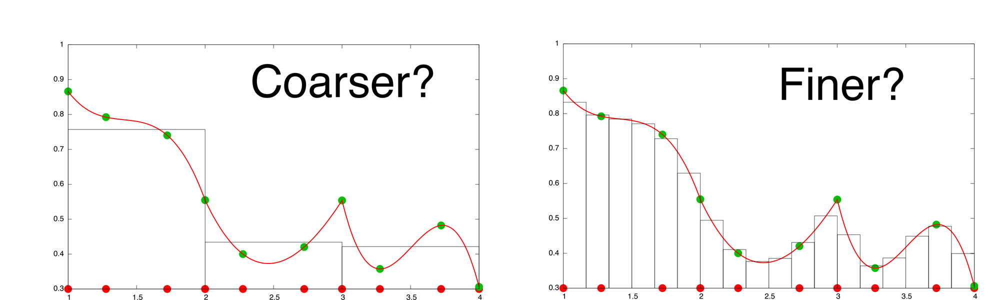
We propose to reconstruct a polynomial $\widehat{\psi}_k(x)$ that satisfies the mass-conservation constraint in all physics grid finite-volumes in element k:

$$\int_{x_{j-1/2}}^{x_{j+1/2}} \widehat{\psi}_k(x)\,dx = \overline{\psi}_j \Delta x,$$

where j=1,..,nc (nc is the number of physics grid finite-volumes in element k). This polynomial is then evaluated at the GLL points to provide physics tendencies to the dynamical core.

Note: If dynamical core uses polynomial order N and nc=N+1 then $\widehat{\psi}_k(x)$ will be identical to the dynamical core Lagrange basis!

What should resolution of physics grid be? nc=N-1?



Reference

Nair, R.D., M.N. Levy and P.H. Lauritzen, 2011: *Emerging numerical methods for atmospheric modeling* Lecture Notes in Computational Science and Engineering, Springer, Vol. 80, pp.251-311.

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