Towards Parameterization of Root-Rock Interaction for the CESM: I. Stochastic Hydraulic Conductivity

Michail D. Vrettas and Inez Fung
University of California, Berkeley - Department of Earth & Planetary Science.

Introduction

The vertical distribution of subsurface moisture and its accessibility for evapotranspiration is a key determinant of the fate of ecosystems and their feedback on the climate system. Observations in coastal Northern California show that water table some **20**m below the surface rises by as much as one meter within days after the first winter rain, suggesting preferential flow paths and water storage in rock fractures that may sustain transpiration through summer dry seasons.

- **Motivation:** The hydrologically active soil column in CLM4 [1], has a globally uniform thickness of **3.8**m. This unrealistic critical assumption is a deficiency of the model and requires further attention in the future development of the model.
- **Goal:** Ultimately, we are interested in developing algorithms to model deep subsurface water dynamics along with its interaction with deep root systems.

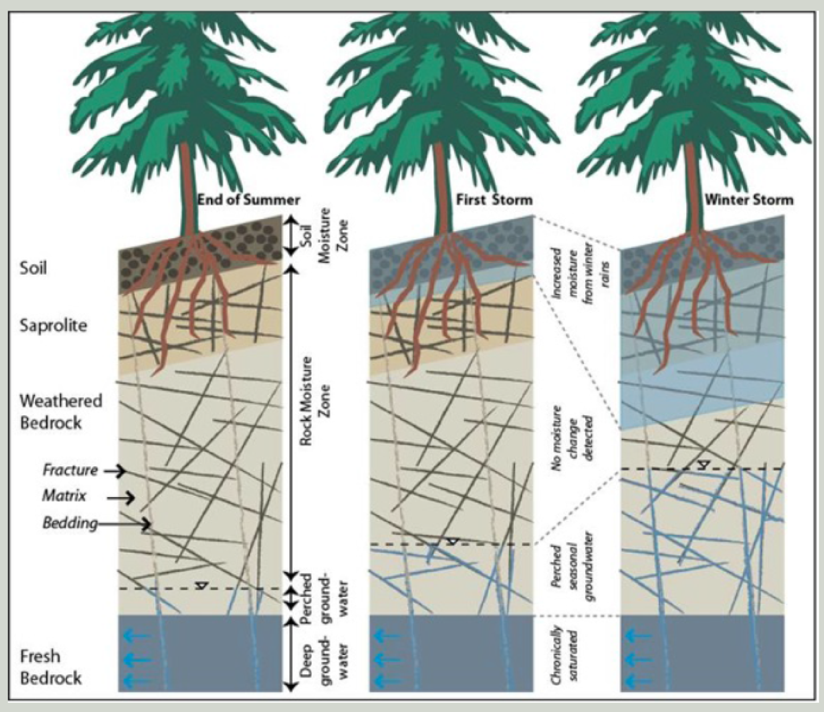

Figure : Illustration of the proposed vertical structure. The heterogeneous subsurface is divided into three layers (soil, saprolite and weathered bedrock), with varying depths.

Governing Equations

Richards' PDE describes the movement of liquids in unsaturated porous media [2]. The pressure head ψ(z,t) form of this equation is given by:

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\right] + S(z,t)\ , \qquad (1)$$

where z is the vertical space dimension [L], t is the time dimension [T], ψ(z,t) is the pressure head (suction) [L], C(ψ) is the specific moisture capacity [1/L], K(ψ) is the unsaturated hydraulic conductivity [L/T] and S(z,t) is a sink/source term [1/T]. In our setting **L : cm** and **T : hrs**. For our experiments we specify the monotonically decreasing vertical $K_{sat}(z)$ and $\theta_{sat}(z)$ (Fig. 2) profiles and apply the following boundary conditions:

- Top Boundary:
$$K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\Big|_{z=0} = q_{rain},$$
- Bottom Boundary:
$$K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\Big|_{z=z_{fb}} = 0.$$

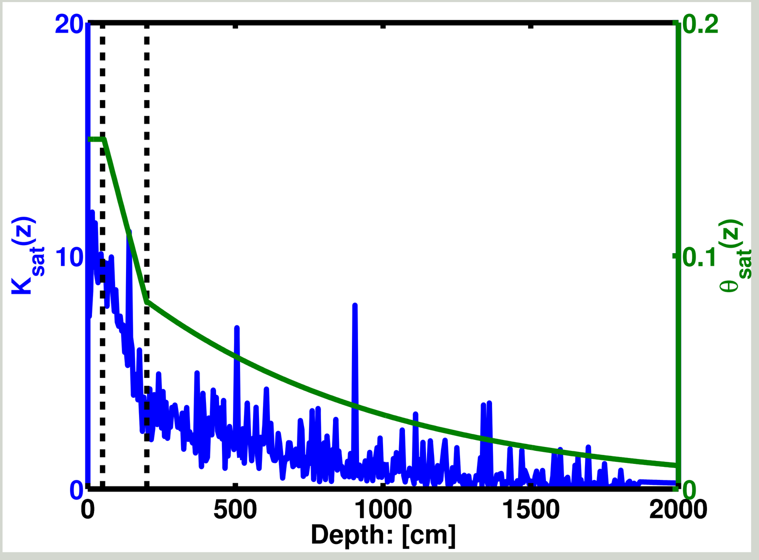

Figure : (2) $K_{sat}(z)$ and $\theta_{sat}(z)$ profiles.

- The soil-water content θ as a function of the pressure head ψ is given by:
$$\theta(\psi) = \theta_{ret} + (\theta_{sat} - \theta_{ret})[1 + (\alpha|\psi|)^n]^{-m}\ , \qquad (2)$$
with $\alpha = 0.0335$, $m = 0.5$, $n = 2$ and θ_{sat}, θ_{ret} are the maximum and minimum values of soil moisture respectively.
- The main parameter of interest here is the **hydraulic conductivity** K(ψ).

New Stochastic Model

Traditional approaches for modeling the hydraulic conductivity K($\cdot$) include the well known van Genuchten model [3]:

$$K(\Theta) = K_{sat}\sqrt{\Theta}[1 - (1 - \Theta^{1/m})^m]^2\ , \text{with } \Theta = \frac{\theta - \theta_{ret}}{\theta_{sat} - \theta_{ret}}\ . \qquad (3)$$

However this model is very slow in capturing the fast response of the water table, with dry initial conditions (as the ones found in California). Instead in this work we propose a new **stochastic** model:

$$K(\Theta) = \Theta^{\lambda}K_{sat}\ , \text{with } K_{sat} \sim \text{LogNormal}(\mu(z), \sigma(\Theta))\ . \qquad (4)$$

We use the stochastic part of the equation to capture the effect of the fractures in the underground that would allow the water to move faster by providing preferential paths with higher conductivity. Here we set $\lambda = 1$.

Data Assimilation Methodology - EnKf

To assimilate the water table data obtained from the `Berkeley HydroWatch Project`, with the model equations (Richards' PDE) we employ an ensemble Kalman filter (EnKf) [4], with perturbed observations.

Datasets: Berkeley HydroWatch Project

- Located on a small (**4000** m^2) steep (**35°**) hill-slope nicknamed 'Rivendell' in the Angelo Coast Range Reserve in the Eel River Watershed, in Northern California, the `Berkeley HydroWatch Project` has drilled **12** wells and uses more than **800** sensors to record the hydrologic status in real time.
- High frequency (less than **30** minutes) data, for nearly five years, shows that the water tables, roughly **18** meters below the surface, can respond in less than **8** hours to the first rains, suggesting very fast flow through macro-pores and fractured rock (e.g. Fig. 1).

Simulations & Results

The following table summarizes the setting for the **EnKf** simulation:

Name :	Z_{tot}	δz	T_{tot}	δt	θ_{ret}	$\theta_{sat,max}$	$K_{sat,max}$	N_{ens}
Value :	2000 cm	5 cm	240 hr	30 min	0.001	0.15	10 cm/hr	50

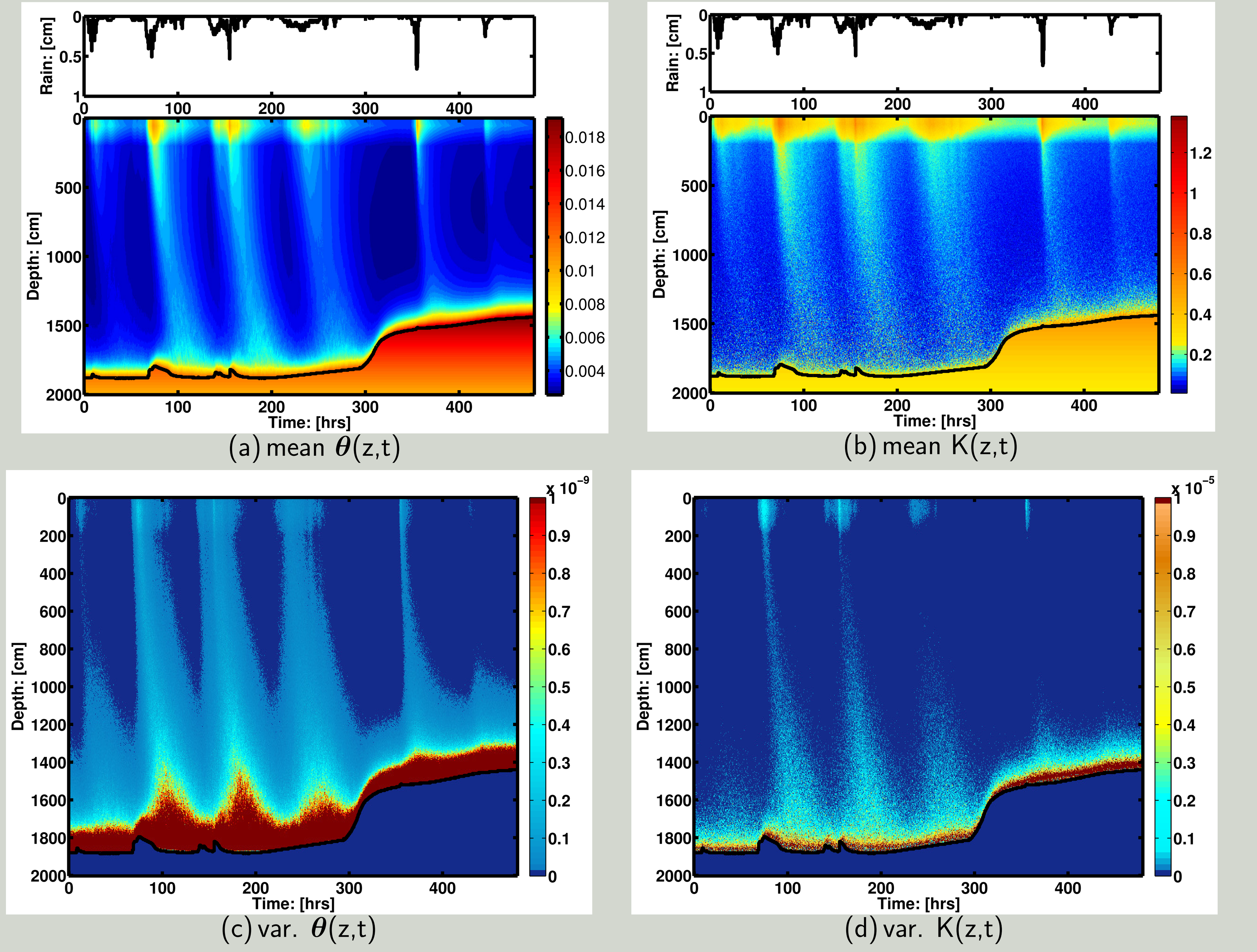

Figure : Ensemble mean for (a) the soil moisture θ(z,t) and (b) hydraulic conductivity K(z,t), for a period of **10** days. Top panels show the precipitation (rainfall in **cm**). Figures (c) and (d) show respectively the ensemble variance of the same quantities. In all figures the observed water table depth (**black** solid line) is contrasted against the estimated results.

- Results show that the stochastic hydraulic model (Eq. 4) can capture the rapid response of the water table depth to rainfall. As expected, the ensemble mean of K(z,t) decreases with depth. Its variance is greatest at depth because of the possible transient flow through rock fractures. With its low permeability, but large thickness, the weathered bedrock layer between **2**m and the water table contains **61.31**% of the soil and rock moisture. This large store of water could be accessible by plants with deep roots and play a crucial role to their survivability through the dry summers.

Future Work

Initial results are encouraging and the next steps include testing this new stochastic approach on data from other sites and ultimately implementing it into the CLM.

Acknowledgements

This work is funded by the NSF, Mathematics and Climate Research Network (DMS-0940272) and the Department of Energy (DE-SC0010857). Part of this research used computational resources of the National Energy Research Scientific Computing center (DE-AC02-05CH11231).

References

[1] K. W. Oleson, et.al., *Technical Description of version 4.0 of the Community Land Model (CLM).* National Center for Atmospheric Research (NCAR), TN-478+STR, 2010.

[2] L. A. Richards, *Capillary conduction of liquids through porous mediums.* Physics, vol. 1, pp.: 318-333, 1931.

[3] M. van Genuchten, *A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils.* Soil Science Society of America, vol. 44, no. 5, pp.: 1-8, 1980.

[4] G. Evensen, *Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics.* Journal of Geophysical Research, vol. 99, no. 10, pp.: 143-162, 1994.

http://vrettasm.weebly.com
{m.vrettas, ifung}@berkeley.edu