Optimal Initial Conditions for Coupling Ice sheet Models to Earth System Models

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Motivation

Methods

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Uncertainty Quantification

Uncertainty in predictions from ice sheet models (e.g. sea-level rise) come from multiple sources, including predominately:

(1) Uncertainties in model forcing - largely related to uncertainties in future climate, which are being explored through emissions-scenario-dependent and perturbed physics ensemble analyses **

(2) Model uncertainties – largely due to uncertainties in initial and boundary conditions **

PISCEES UQ will focus on the latter, specifically:

(i) Constraining uncertain initial and boundary condition parameters
(ii) Estimating parameter uncertainties using a combination of intrusive (adjoint) and non-intrusive (sampling) approaches **
(iii) Forward propagation of input parameter uncertainties to assign uncertainties to ice sheet model outputs of interest

*** See talks by: Heimbach et al., Jackson et al. ***
& poster by Salinger et al. (#43, room 1)

Motivation

Existing ice sheet model initialization methods do not couple smoothly with realistic climate forcing (from models or observations).

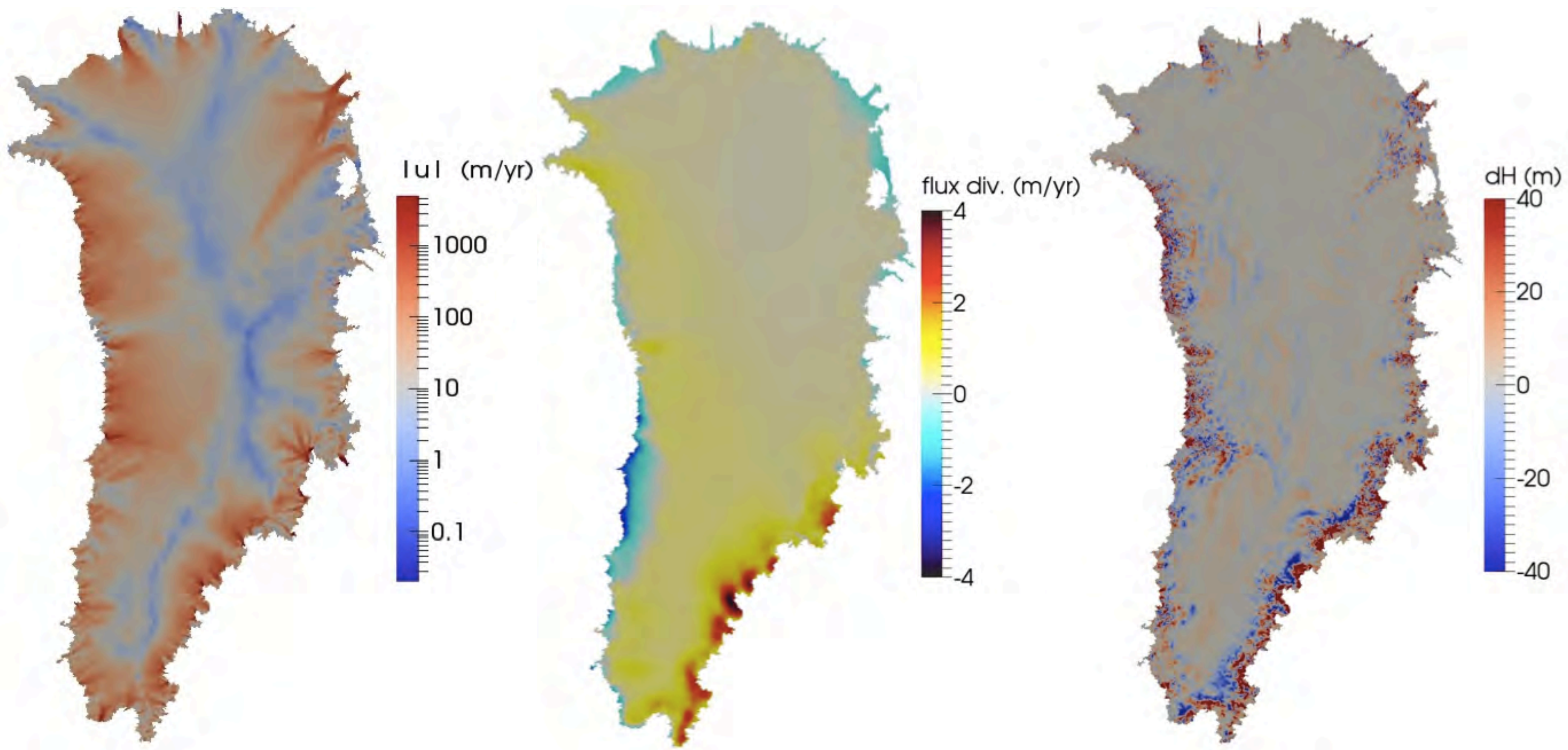
1. Spin-up: initial condition is consistent with climate forcing & long-term transients, but difficult (impossible?) to combine with the goal of closely matching present-day obs. (geometry, flux, etc.)

2. Optimization: fix model geom. to obs., tune model parameters (e.g., basal sliding coeff.) to provide optimal match to observed vels.

Method (1) is impractical because (i) ice dynamic response on 10^1-10^2 yr timescales is very strong function of initial geom. and vel., and (ii) spin-up of appropriate duration (10^4-10^5 yr) not practical for high-res., next-gen. ice sheet models.

Method (2) provides good match to present-day obs. but generally leads to unphysical “shock” when coupling to SMB from climate model.

Motivation



At equilibrium, the SMB is balanced by the flux divergence

SMB = surface mass balance = ice accumulation less melting & sublimation

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Optimization Problem

To avoid non-physical shocks when coupling ice sheet models to ESMs, the model flux divergence must be balanced by the model SMB.

$$\frac{\partial H}{\partial t} = \underbrace{-\mathrm{div}\,(\mathbf{U}H)}_{\text{flux divergence}} + \overbrace{\tau_s}^{\text{surface mass balance}}, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz$$

At equilibrium: $\mathrm{div}\,(\mathbf{U}H) = \tau_s$

Basal boundary condition: $(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad on \quad \Gamma_\beta$

At the same time, we want the initial model geometry and velocity field to match present-day observations (obtained by optim. "beta" field)

Solution: PDE-constrained optimization with constraints on both velocity (commonly applied) *and* flux divergence (novel), *additionally* accounting for uncertainties in ice sheet geometry (thickness)

Optimization Problem: Find β and H that minimize the cost functional $\mathcal{J}$:

Optimization Problem

Find β , H that minimize the objective functional

$$\mathcal{J}(\beta, H) = \int_\Sigma \frac{1}{2\sigma_u^2} |\boldsymbol{u} - \boldsymbol{u}^{\mathrm{obs}}|^2 \, ds$$

surface velocity mismatch — *Common*

$$+ \int_\Sigma \frac{1}{2\sigma_\tau^2} |\nabla \cdot (\boldsymbol{U} H) - \tau_s|^2 \, ds$$

SMB mismatch

$$+ \int_\Sigma \frac{1}{2\sigma_H^2} |H - H^{\mathrm{obs}}|^2 \, ds$$

thickness mismatch

Novel

$$+\mathcal{R}(\beta, H)$$

Regularization terms

subject to ice-sheet model equations

(high-order approximation of nonlinear Stokes equations).

Perego, Price, Stadler (*JGR Earth Surface*, in review)

Numerical / Computational Details

Fwd model (PDE constraint):

- 1st-order Stokes approximation[1] (FELIX prototype model)
- FEM discretization
- variable resolution, tetrahedral mesh (min. res. ~4 km)

Numerical method:

- Quasi-Newton using LBFGS for cost function minimization
- cost function gradients provided by fwd model adjoint [1]

Software Frameworks:

- *LifeV* FEM library
- *Trilinos:*
  - NOX – Newton nolinear solver in fwd model
  - AztecOO – PCG linear solver in fwd model
  - ROL – Rapid Optimization Library (ROL) for LBFGS

[1] Perego et al. (2012)

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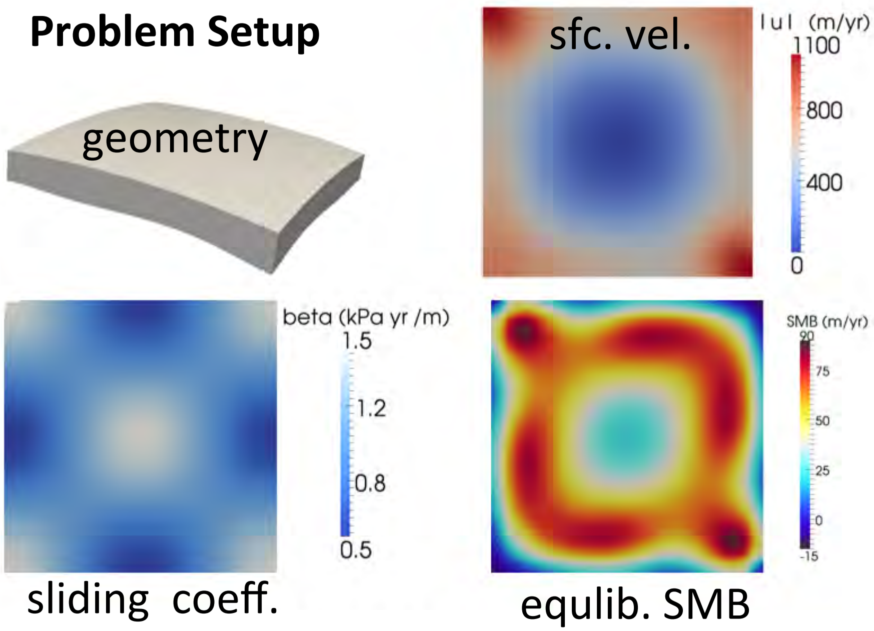
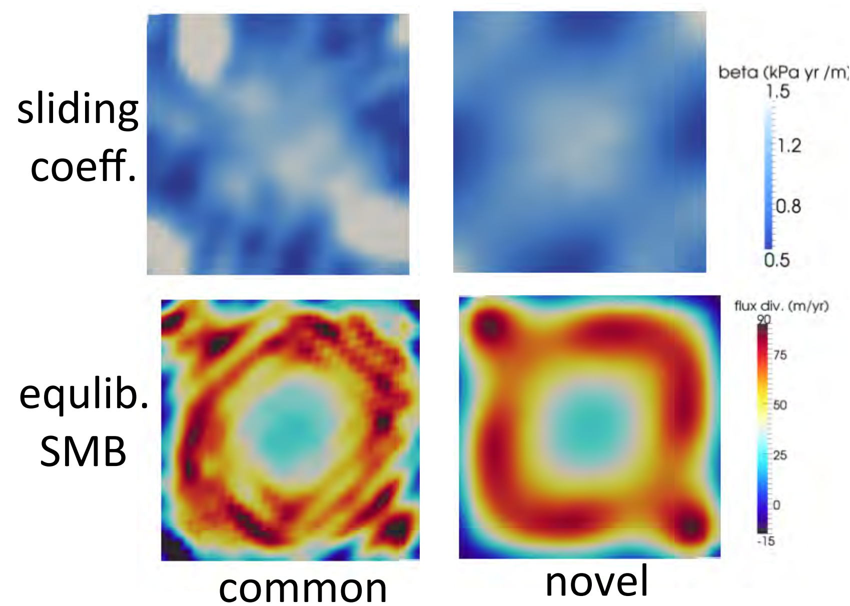
Synthetic Application

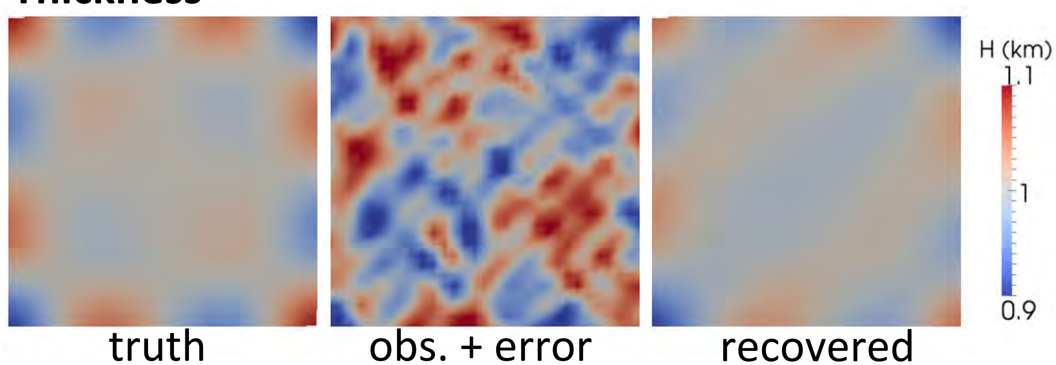
Problem Setup



Inversion Results



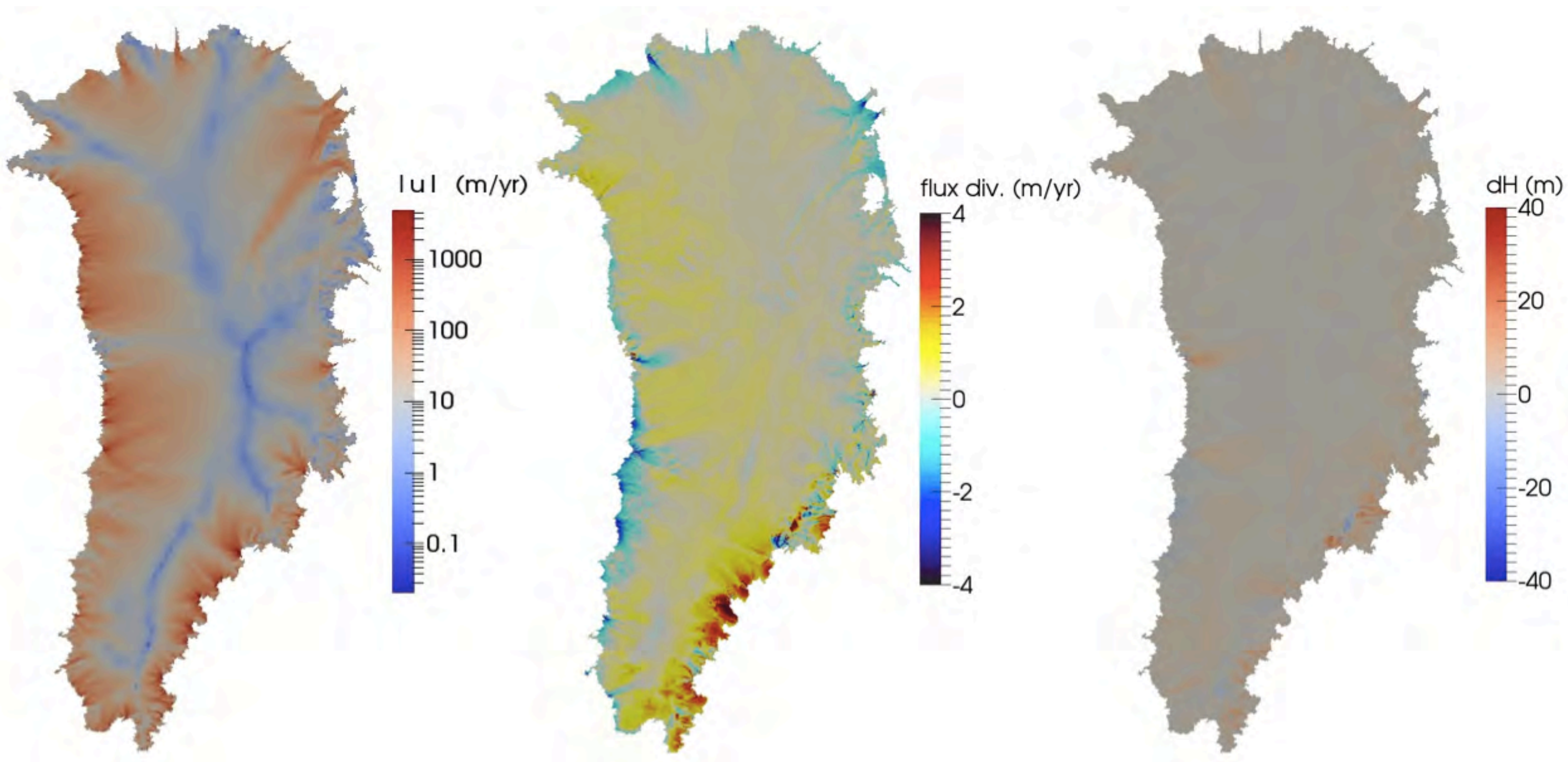
Thickness



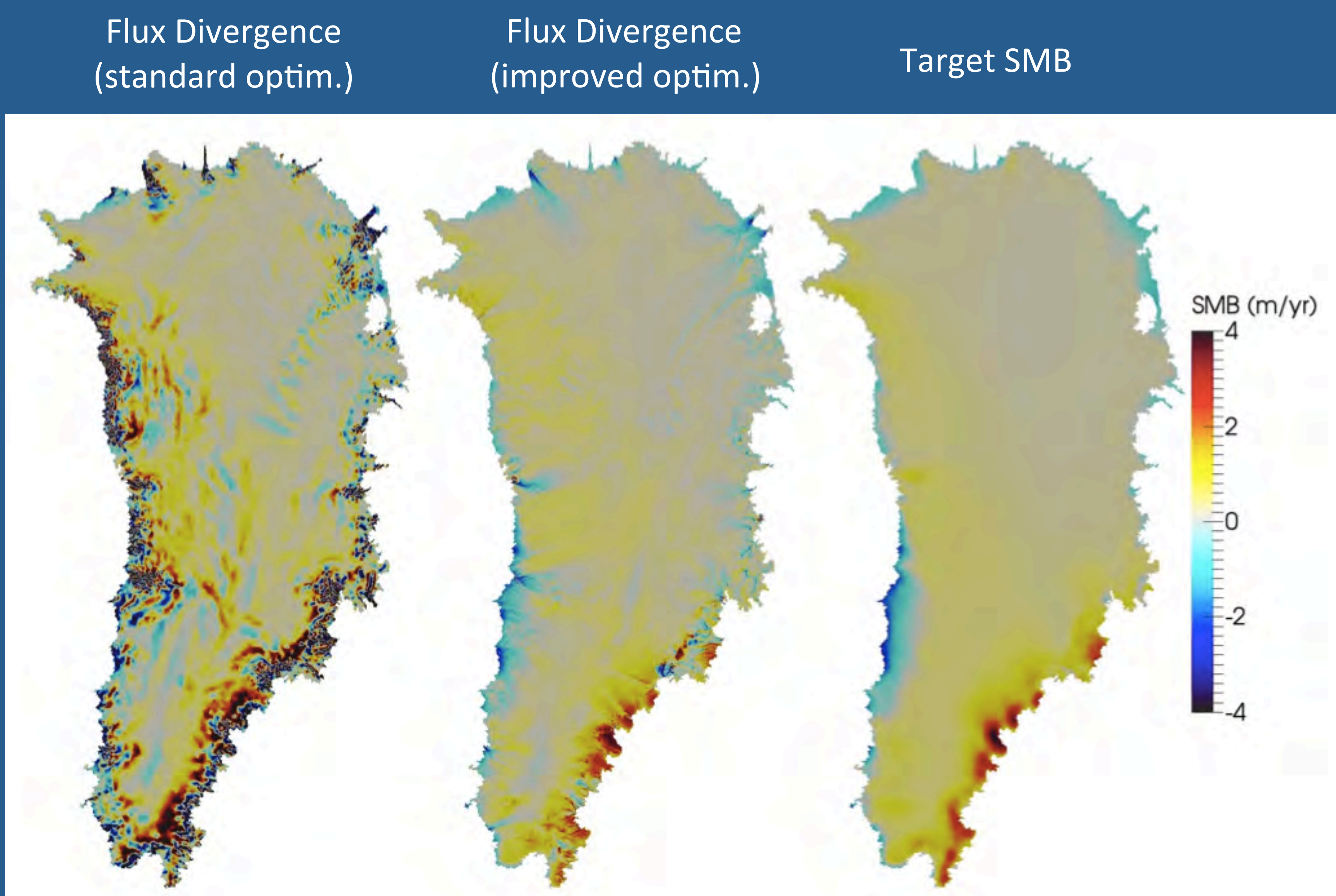
- Common and novel method succeed in recovering sfc. vel.
- Common method does not allow for equilib. SMB constraint
- Common method has difficulty with uncertain thickness data
- Novel method addresses both of these failings

error in recovered velocity

Realistic Application



Realistic Application

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Demonstration of promising optimization framework for generating realistic ice sheet model initial conditions that also couple smoothly to climate models

… but how does this relate to UQ?

Next Steps:

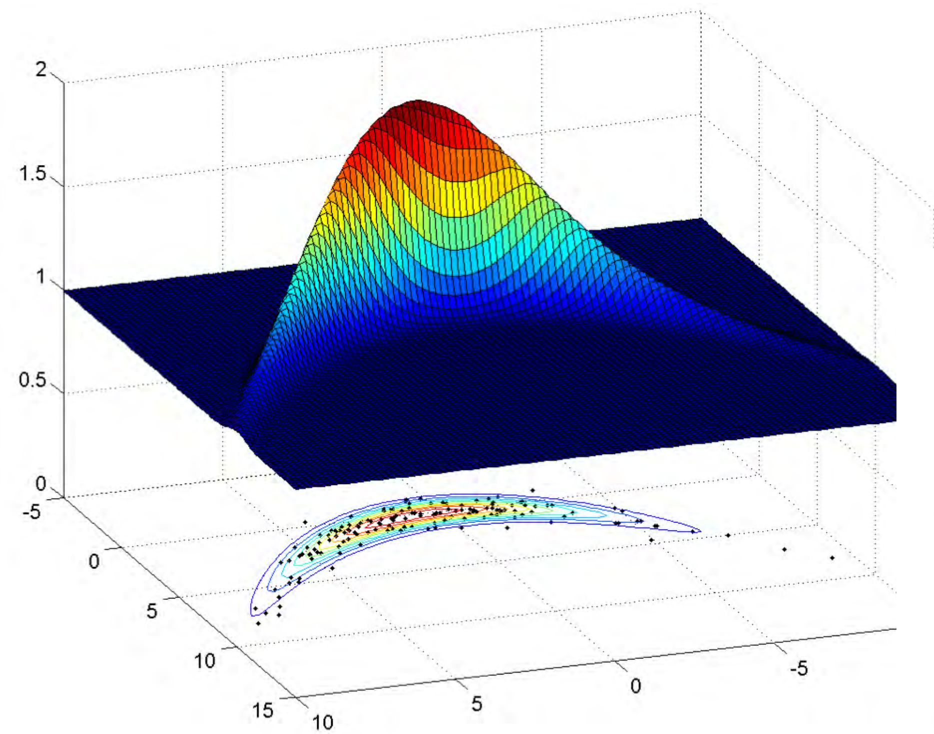
Use approximate Hessian to aid in sampling of posterior parameter distributions ** (in addition to finding the MAP point, as shown here)

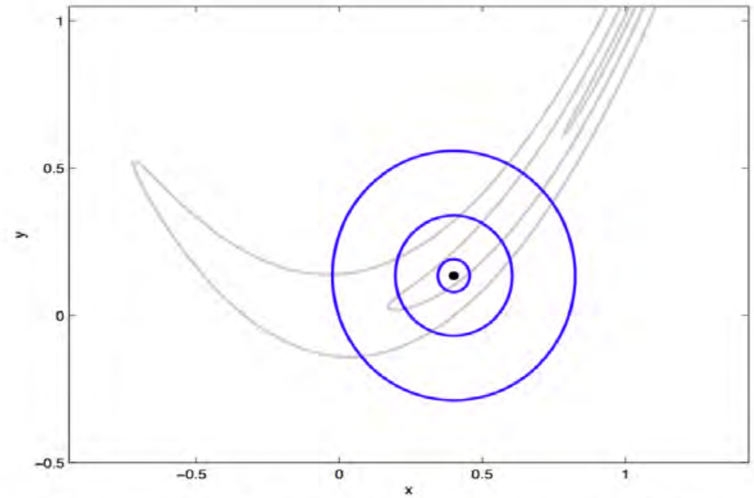
Conduct fwd simulations from these approximate distributions in order to assess resulting uncertainties on model outputs of interest (e.g. ice sheet mass loss and sea-level rise)

** Requires a combination of intrusive (adjoint) and non-intrusive (sampling) methods, including ***human*** and ***software*** support for both

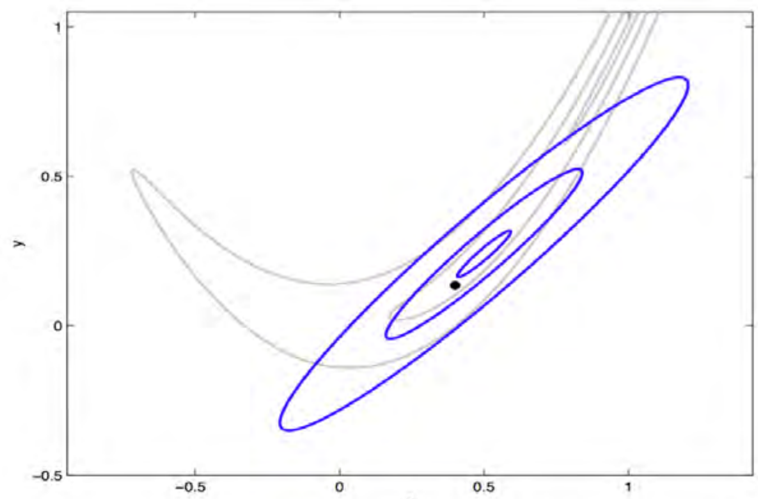
Future Work

Use of approximate Hessian to increase acceptance rate of MCMC sampling of posterior parameter distribution





The contours of the random walk proposal function overlayed on the Rosenbrock contours.

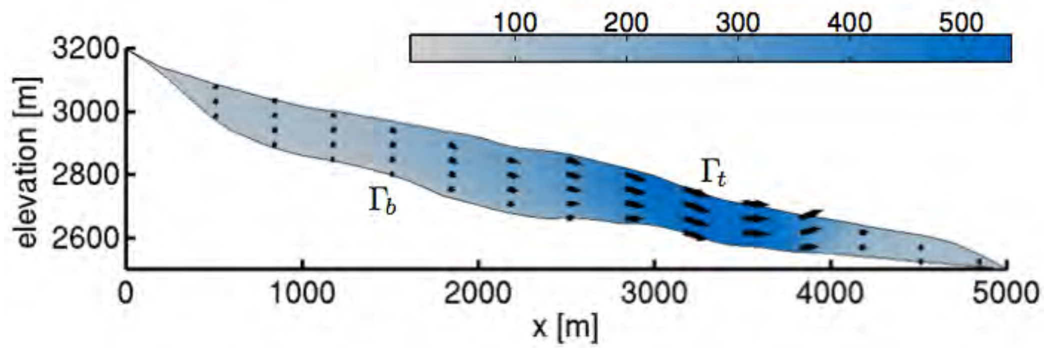


The contours of the stochastic Newton method proposal function overlayed on the Rosenbrock contours.

Figures courtesy of G. Stadler

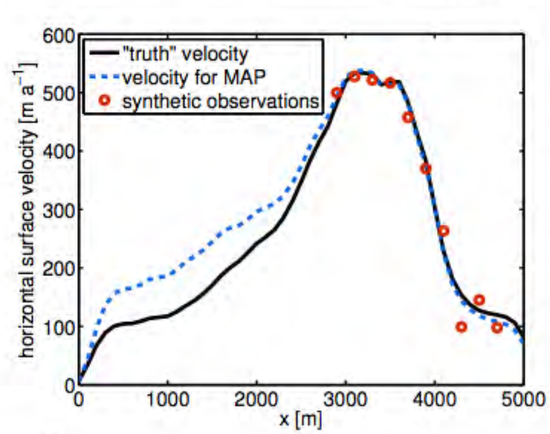
Future Work

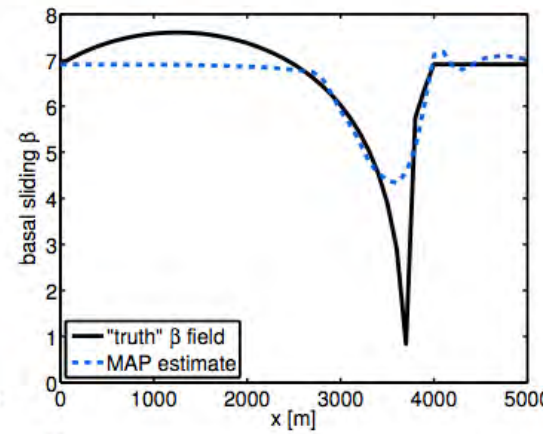
Modeled 2d (x,z) velocity field with basal sliding coefficients tuned to match observations



Left: modeled (blue) and "true" (blk) surface velocity profile and synthetic observations (red)

Right: MAP estimate (blue) and "true" basal sliding coefficient (blk)





Left: prior estimate for basal sliding coefficient distribution

Right: posterior coefficient distribution obtained using Hessian-informed MCMC sampling