# Time-Stepping Optimized for Ocean and Atmosphere Models

### Objective

- Develop a new class of time-stepping methods optimized to take time-steps as large as possible on the shallow water equations (SWEs), which are foundational to ocean and atmosphere models.

### Approach

- Alter an existing time-stepping scheme to use a weighted average of thickness data from both forward and backward in time (a forward-backward average) to advance the momentum equation.
- Optimize the weights of this forward-backward average to maximize the theoretical bound on the time-step when the scheme is applied to the SWEs.
- Test the scheme (FB-RK(3,2)) on both linear and nonlinear SW test cases and record the speedup as compared to a comparable, popular method often used in SW models (SSPRK3).

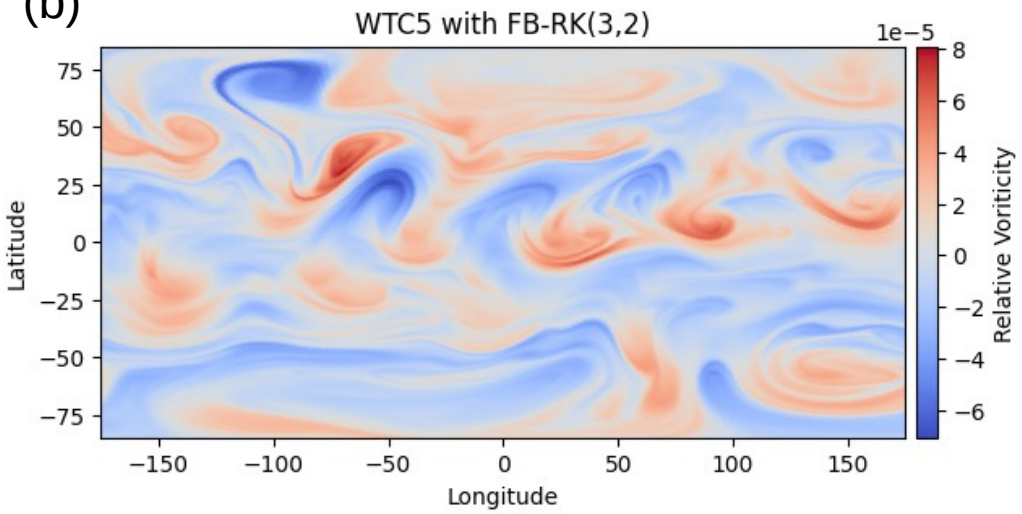
### Impact

- The FB-RK(3,2) provides a speedup of 2.8 times (in terms of wall-clock time) on a linear test case, and speedups between 1.6 and 2.2 in fully nonlinear cases without effecting solution quality.
- The new scheme could be adapted to be a barotropic solver in a split barotropic/baroclinic ocean model to drastically reduce the cost of what is often the most expensive part of the model.

(a)

| Maximum Admittable Time-Steps in Seconds | | |
|---|---|---|
| | Old Method | New Method |
| Test Case | SSPRK3 | FB-RK(3,2) |
| Quasi-Linear | 515 | 1445 |
| speedup | | 2.81 |
| Barotropic Jet | 110 | 195 |
| speedup | | 1.77 |
| Williamson TC 2 | 220 | 370 |
| speedup | | 1.68 |
| Williamson TC 4 | 110 | 240 |
| speedup | | 2.18 |
| Williamson TC 5 | 145 | 310 |
| speedup | | 2.14 |

(b)




**FB-RK(3,2) outperforms SSPRK3 in terms of time-step on multiple test cases (a) and retains solution quality (b), error compared to SSPRK3 is ~10e-8.**


Jeremy R. Lilly, Darren Engwirda, Giacomo Capodaglio, Robert L. Higdon, and Mark R. Petersen. “CFL Optimized Forward–Backward Runge–Kutta Schemes for the Shallow-Water Equations”. In: Monthly Weather Review 151.12 (Dec. 2023), pp. 3191–3208. issn: 1520-0493, 0027-0644. doi: 10.1175/MWR-D-23-0113.1.